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SCENARIOS AND SIGNALS OF VERY HEAVY NEUTRINOS

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• PRELIMINARY REMARKS

Neutrinos are very special elementary fermions. Their uniqueness lies in not having any electromagnetic charge. This enables massive neutrinos to be either Dirac fermions with distinct particles and antiparticles or, giving up lepton conservation, self-conjugate Majorana fermions. (The distinction becomes meangingless in the massless case when a Dirac fermion simply becomes a linear superposition of two Majorana ones.) For quite some time now there have been speculations [1,2] on the existence of very heavy neutrinos. This is the subject of my talk.

I shall discuss the interest in the properties of very heavy neutrinos (generically denoted as N) lying in the mass range

$$45 \text{ GeV} < M_N < \mathcal{O}\ (\text{TeV}).$$

The lower bound is dictated by the lack of observation at LEP 1 of the pair-production of any such neutrinos. The upper bound is flexible upto a factor of a few owing to possible mixing angle uncertainties. This bound stems from the requirement [3] of perturbative unitarity in the neutrino pair-production amplitude from the initial state of two longitudinally polarized W or Z bosons.

It is convenient to consider alongwith N the charge-conjugated field $N^C \equiv (\bar{N}C)^T$ where C is the charge-conjugation matrix in spinor space. For Dirac neutrinos $N \neq N^C$ whereas Majorana ones obey the condition $N = N^C$. The mass term for a heavy Dirac neutrino can be written as

$$\mathcal{L}^D_{mass} = -M^D_N \bar{N} N = -M^D_N (\bar{N}_L N_R + \bar{N}_R N_L), \tag{1}$$

where $N_{L,R} \equiv \frac{1}{2}(1 \mp \gamma_5)N$. There are four independent chiral components in this case: N_L, N_R, ${N_L}^C = N^C_R$ and ${N_R}^C = N^C_L$. In contrast, a massive Majorana neutrino admits a term such as

$$-\frac{1}{2} M^M_N \bar{N}^C N + h.c. = -\frac{1}{2} M^M_N (N^T_L C^{-1} N_L + N^T_R C^{-1} N_R) + h.c.,$$

where use has been made of the relation $\gamma^0 C^\star \gamma^0 = C^{-1}$ in the last step. More generally, Lorentz invariance allows separate Majorana masses $M^M_{L,R}$ for $N_{L,R}$ so that one can take

$$\mathcal{L}^M_{mass} = -\frac{1}{2} M^M_L N^T_L C^{-1} N_L - \frac{1}{2} M^M_R N^T_R C^{-1} N_R + h.c. \tag{2}$$

The most general mass term would be a sum of (1) and (2).

If N_L transforms as a doublet and N_R as a singlet of the electroweak $SU(2) \times U(1)$ gauge group, $M^M_L (M^M_R)$ would have to arise from the VEV of a Higgs field transforming as a triplet (singlet). Since a triplet representation usually has problems with a unit value of the ρ-parameter, it is customary to take $M_L = 0$ with the caveat that high scale physics may induce a tiny M_L. On the other hand, a Higgs singlet – being outside the electroweak gauge theory – could only be a relic of high scale physics so that one expects $M_R = M \gg M_W$. The general mass term for N can then be written in matrix form as

$$\mathcal{L}_{mass} = -\frac{1}{2} (\overline{N_L}\ \overline{{N_R}^C}) \begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix} \begin{pmatrix} {N_L}^C \\ N_R \end{pmatrix} + h.c. \tag{3}$$

In the seesaw [4] mass-matrix of (3) the off-diagonal element m_D is the Dirac mass. It may be expected to be typically of the order of the known charged fermion masses of the concerned

family in the Standard Model. Of the two eigenvalues of the seesaw mass matrix, one ($\sim M$) is therefore expected to be heavy and the other ($\sim m_D^2/M$) light. In this simplest of seesaws the physical particles are Majorana fermions, but more complicated seesaws exist [5] where they are of the Dirac type. Furthermore, arguments have been given [6] that the seesaw mechanism may be induced radiatively at the electroweak 1-loop level. Thus the seesaw formula for a light neutrino might in fact be $m_\nu \sim \pi^{-1}\alpha_W m_D^2/M$, where α_W is the weak fine structure constant. In this case a physical electron neutrino mass $\sim 10^{-2}$ eV (of interest to the solar neutrino puzzle) and a Dirac mass $\mathcal{O}$ (MeV) would suggest a heavy righthanded neutrino mass $M \sim 10^2 - 10^3$ GeV which may be within reach of production and detection in the forthcoming colliders.

• THEORETICAL SCENARIOS

There exist a number of scenarios in which such very heavy neutrinos are expected to occur. We outline a few.

#1. *Fourth generation model* – This has been proposed by Hill and Paschos [7] and has a heavy charged lepton ℓ and a neutrino N making a fourth replica of the existing three generations. Thus one has a left-chiral doublet and two right-chiral singlets:

$$\begin{pmatrix} N \\ \ell^- \end{pmatrix}_L, \quad \ell^-_R, \quad N_R.$$

Existing LEP constraints from Z-decay simply require that $M_N > \frac{1}{2}M_Z$. One need not have N_R, but then one would be forced to invoke a lepton-number violating Majorana mass term. This model has been shown to be natural in terms of flavor democracy [8] which invokes a permutation symmetry in the 4×4 fermion mass-matrix.

#2. *Left-right symmetric model* – The simplest such model employs the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ and contains one left-chiral and one right-chiral lepton doublet per generation:

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L, \quad \begin{pmatrix} N \\ e \end{pmatrix}_R.$$

These describe a very heavy and a very light physical neutrino. References to discussions of their phenomenology can be found in [9].

The very heavy neutrino, described in the above two scenarios, can be searched for in LEP 200 via the reaction $e^+e^- \longrightarrow N\overline{N}$. This process can proceed both by Z-exchange in the s-channel and by W-exchange in the t-channel, provided the N-mass is less than a 100 GeV. For a higher mass, one would need to wait for the Next Linear Collider (NLC) where picobarn level cross sections are predicted for $\sqrt{s} \sim 300$ GeV. It can, in principle, be looked for by exploiting its mixing with ν_e via the production mode $ep \to N +$ 'anything' at HERA, but the estimated cross sections [10] look impossibly small.

#3. *Pure singlet model* – In this case there is an extra right-chiral singlet heavy neutrino N_R for each generation. Thus, for the first, one has

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L, \ e_R, \ N_R.$$

N_R can have a large Majorana mass and it is possible to arrange a seesaw mass-matrix between ν_L and N_R. The main importance of this type of a model is that N can be produced singly in e^+e^- or ep collision, as detailed later.

#4. *E_6-based models* – The grand unifying gauge group E_6 is very popular among model builders starting with the $E_8 \times E_8$ superstring. The matter fields in an E_6 GUT, arising from the topological breakdown of one of the E_8's, belong to the **27** dimensional representation of E_6. This multiplet accommodates three extra heavy neutrinos [11], their masses depending on various symmetry-breaking scales in the breakdown chain $E_6 \to SU(3)_C \times SU(2)_L \times U(1)_Y$. Two of these neutrinos are singlets under $SU(2)_L \times U(1)_Y$. The third, transforming as part of a vectorial doublet with respect to weak isospin, directly couples to W and Z. The others too can mix with the usual very light neutrinos ($\nu_\ell, \ell = e, \mu, \tau$) and develop couplings to the weak bosons.

#5. *Supersymmetric preon models* – These [12] also contain the very heavy neutrinos of the kind discussed in E_6-based scenarios. Additional ones may be possible but the detailed phenomenology has not been worked out.

• COSMOLOGICAL CONSTRAINTS

We shall discuss these under three headings.

#1. *Stable very heavy neutinos* – The discussion of constraints on the properties of very heavy neutrinos from early universe considerations depends on whether those neutrinos are stable or unstable. In the former situation the universe today would be pervaded by a sea of such objects. We shall keep in mind two generic cases: (1) a stable lefthanded N transforming like an $SU(2)_L$ doublet [7] with a universal four fermion coupling characterized by the Fermi constant G_F; (2) a stable nonstandard $SU(2)_L$-singlet N (say righthanded), with a four-fermion coupling of at least milliweak strength ($\gtrsim 10^{-3}\ G_F$) which is induced by some kind of mixing.

There is an upper bound [13] on the sum of the masses of stable standard neutrinos from the requirement of not over-closing the universe through excess mass-density. However, the commonly written form, i.e.

$$\sum_\nu m_\nu < 100h^2\ eV,$$

where $h \equiv$ (Hubble constant/100 $km\ s^{-1}\ Mpc^{-1}$), is inapplicable to N if its mass exceeds 2 GeV. Such heavy neutrinos equilibriate in the early universe by pair-creation and annihilation. These processes continue to occur till the universe cools below their freeze-out temperature T_f at which point they go out of chemical equilibrium. The argument for the above inequality then needs to be reformulated in terms of statistical mechanics and yields [14]

$$M_N e^{-M_N/(kT_f)} < 100h^2\ eV, \tag{4}$$

k being the Boltzmann constant. The freeze out temperature for very heavy neutrinos has been estimated to be [15] $T_f \sim M_N/(20k)$. Thus (4) is easily satisfied for a Dirac mass in excess of 2 GeV.

Figure 1: Cosmological constraints on stable heavy neutrino masses.

Even in the post-freeze out era, however, the reaction $N\overline{N} \to e^+e^-$ can continue to take place. Subsequently, the produced e^+e^- pair can annihilate into two photons. If unrestricted, this process will generate far too many photons and lead to an unacceptably high value of the entropy density of the universe. The necessary restriction is

$$M_N (T/T_\gamma)^3 < 100h^2 \ eV, \tag{5}$$

where T_γ is the temperature of the cosmic microwave background radiation while T is the temperature that would have accrued to the sea of N's, if N were massless. When (5) is applied to case (1), the restriction $M_N < 3$ TeV follows [16], while, for case (2), the corresponding result is $M_N < 10$ TeV for $G/G_F \sim 10^{-3}$. Studying the latter case in more detail [17], Olive and Turner have obtained quantitative restrictions on M_N as a function of G_F/G as shown in Fig. 1 for Dirac and Majorana N's.

Additional constraints on the masses of very heavy stable neutrinos have emerged [18] from dark matter search experiments. The annihilation of $N\overline{N}$ into charged particles is exploited for this purpose by seeking to detect the latter. The restrictions, consequent upon the lack of such detection, depend on the assumptions made in calculating the flux of N's. There are two possible approaches. (a) If one assumes that the galactic halo is composed solely of such stable very heavy neutrinos, the constraints obtained on their masses, for the Dirac and Majorana cases respectively, are:

$$M_N^D < 6 \text{ GeV or } M_N^D > 300 \text{ GeV},$$
$$M_N^M < 24 \text{ GeV or } M_N^M > 300 \text{ GeV}.$$

(b) On the other hand, if the flux is calculated by considering the relic abundance of N's taking such annihilation processes as $N\overline{N} \rightarrow f\overline{f}, W^+W^-, ZZ$ (f any quark or lepton) into account, the corresponding results become

$$M_N^D < 6 \text{ GeV or } 40 \text{ GeV } < M_N^D < 48 \text{ GeV},$$
$$M_N^M < 24 \text{ GeV or } 38 \text{ GeV } < M_N^M < 52 \text{ GeV}.$$

#2. *Unstable very heavy neutrinos* – The possible channels for the decay of an unstable N, that have been considered, are : $N \rightarrow \nu\gamma$ and $N \rightarrow \nu\ell^+\ell^-$, ℓ being a charged lepton. If N is very massive and decays late in the evolutionary history of the universe, the decay products will make a significant contribution to the energy density of the universe, albeit redshifted by the Hubble expansion. This can yield any meaningful constraint [19] only if the lifetime of N is greater than a second or so. For a highly unstable N, which is of interest to accelerator-based particle physicists, there are practically no constraints from cosmology.

#3. *Leptogenesis* – The existence of a massive Majorana neutrino N is perforce in contradiction with lepton conservation since the mass term acts as a lepton number violating operator. Thus the presence of such particles would mean a lepton asymmetric early universe in which leptogenesis occurred at the (GUT) timescales when N acquired its mass. However, during the electroweak phase transition, sphaleron-induced baryon and lepton nonconserving (but $B - L$ preserving) processes at temperatures between about 100 GeV and 1 TeV would convert this leptonic asymmetry into a baryonic one. This is a viable [20] scenario (i.e. the sphaleron processes can remain in thermal equilibrium for the requisite period) for a whole range of heavy neutrino masses from about 1 TeV to 10^{12} GeV. It has also been demonstrated [21] that, even if the conservation of the full $B - L$ is invalid, that of $\frac{1}{3}B - L_i$ – where i refers to *any* leptonic type – is sufficient for the mechanism to go through.

• COUPLINGS OF VERY HEAVY NEUTRINOS

There are far too many model-independent possibilities in the pattern of couplings of such very heavy neutrinos to the known elementary particles. We try to adopt a generic approach following [22]. Let us assume that, on account of mixing with ν_ℓ, N develops a charged current coupling to $W\ell$ and neutral current couplings to NZ as well as $\nu_\ell Z$ – as shown in Fig. 2. Here ξ is a small

Figure 2: Heavy neutrino couplings.

seesaw mixing factor (hopefully $\gtrsim 10^{-3}$) and $V_{N\ell}$ is a Kobayashi-Maskawa type matrix element. (It may be noted that the model of [7] cannot be covered by this since there is no $ZN\bar{\nu}_\ell$ vertex there and no mixing factor in the $ZN\bar{N}$ coupling). We are also obliged to choose $M_N > M_Z$, otherwise – for $\xi > 10^{-3}$ – the decays $Z \to M\bar{\nu}_\ell, \bar{N}\nu_\ell$ would have already been seen at LEP.

We come to the decays of N. First, consider the case when N is a Dirac particle. Now, for the charged current mode

$$\Gamma(N \to \ell^+ W^+) = \Gamma(\bar{N} \to \ell^+ W^-) = \frac{|\xi V_{N\ell}|^2}{8\sqrt{2}\pi} \frac{G_F}{M_N^3} (M_N^2 + 2M_W^2)(M_N^2 - M_W^2)^2. \tag{6}$$

Contrariwise, for the neutral current mode

$$\Gamma(N \to \nu_\ell Z) = \Gamma(\bar{N} \to \bar{\nu}_\ell Z) = \frac{|\xi|^2}{8\sqrt{2}\pi} \frac{G_F}{M_N^3} (M_N^2 + 2M_Z^2)(M_N^2 - M_Z^2)^2. \tag{7}$$

The equality of the N and $\bar{N}$ partial widths in (6) and (7) follows from CP-invariance. The charged current mode is less dominant than the neutral current one since the latter does not have the small $|V_{N\ell}|^2$ factor. For a Majorana heavy neutrino, N and its antiparticle are identical. Now one simply has $\Gamma(N \to \ell^- W^+) = \Gamma(N \to \ell^+ W^-)$ and $\Gamma(N \to \nu_\ell Z) = \Gamma(N \to \bar{\nu}_\ell Z)$, with the corresponding expressions still given by (6) and (7) respectively. Hence the lifetime of N gets halved as compared with a Dirac N. In either case, for $M_N \gg M_{W,Z}$ and $\xi \gtrsim 10^{-3}$, the mean free path is $\ll 1$ cm. Thus, if produced in the laboratory, such an N will decay within the detector.

Though the neutral current induced decay is the dominant mode, the charged current mediated one ($N \to \ell W$) can provide the cleanest signals for detection. The W can decay into two jets so that $\ell(2j)$ is the detectable final state configuration. For the Dirac case and with a pair-produced $N\bar{N}$, one would have the hard signal $\ell^+ e^{'-} (4j)$ where ℓ and ℓ' need not be the same. (Of course, one would have to tackle the severe background from the semileptonic decays of top-antitop pairs). If a pair of Majorana N's gets prodeuced, we can have three possibilities : $\ell^+ e^{'-} (4j)$, $\ell^+ e^{'+} (4j)$ and $\ell^- e^{'-} (4j)$. While these are characteristic signals, one cannot exclude a very heavy neutrino in

the relevant mass-range simply by failing to observe them. This is because certain models allow [23] the dominant decay $N \to \nu J$ where J is a very light pseudo-Goldstone boson like a Majoron. The experimental unobservability of this decay channel would make it harder to discover N.

• PRODUCTION MECHANISMS

First, we take up the production of single N's. In an e^+e^- collider this can be done through the processes $e^+e^- \to N\bar{\nu}_\ell, \bar{N}\nu_\ell$. These go via the Feynman diagrams of Fig. 3. Asymptotically, for a large CM energy $\sqrt{s}$, the cross section is approximately $\pi^{-1}G_F^2 M_W^2 |V_{\ell N}\xi|^2$.

Figure 3: Diagrams for single N or $\bar{N}$ production in e^+e^- collision

Note that, for a Majorana N, there are only three diagrams since (c) and (d) become one and the same. In any event, the Z-mediated part contributes only about 2% of the total cross section. Thus it is a good approximation to take only the W-mediated part. Typically, a fraction of a picobarn is expected at LEP 200 as shown in Fig. 4, where the production cross section [22] has been plotted against $\sqrt{s}$ as well as against the heavy neutrino mass M_N. Coming to electroproduction $e^-p \to NX$, say at HERA, the cross section is shown against M_N for various values of $\sqrt{s}$. Of course, the signal (Fig. 5) will depend on whether the N decays into ℓW or $\nu_\ell Z$, but $\ell^-(2j)\,\not{E}_T$, $\ell^+\ell^-\,\not{E}_T$, and $\ell^+e'^-\,\not{E}_T$ are possible signal configurations.

Figure 4: Cross sections for $e^+e^- \to N\bar{\nu}_e$ at LEP 200 as functions of M_N and $\sqrt{S}$.

Figure 5: Cross sections for $\bar{e}p \to NX$ at HERA as a function of M_N.

Next, we come to pair-production. This can be attained through the $ZN\bar{N}$ coupling which is perhaps less model-dependent than the $ZN\bar{\nu}$ one. We put a generic mixing factor χ to cover the cases where N is an $SU(2)_L \times U(1)_Y$ singlet. (For a regular fourth generation heavy neutrino, χ is unity). The cross section for $e^+e^- \to Z^\star \to N\bar{N}$ (Fig. 6) can be calculated [11] to be

$$\sigma = \frac{G_F^2 s}{24\pi}\left(1 - \frac{4M_N^2}{s}\right)^{1/2}\left(1 - \frac{M_N^2}{s}\right)\left(\frac{M_Z^2}{M_Z^2 + s}\right)|\chi|^2(1 - 4x_W + 8x_W^2). \tag{8}$$

Figure 6: Lowest order diagram for $e^+e^- \to N\bar{N}$

In the high-energy limit when $s \gg M_Z^2$, the RHS of (8) goes as $2.5 \times 10^{-2}|\chi|^2$ (pb/s in TeV^2) which is about $0.6|\chi|^2$ pb for $\sqrt{s} = 200$ GeV at LEP 200. Similar considerations hold for the Drell-Yan type of production process $q\bar{q} \to Z^\star \to N\bar{N}$ in a hadron collider. One problem with the cross section of (8) is the rapid fall off at large s which drastically reduces the cross section at supercollider energies $\gtrsim 1$ TeV.

We shall discuss an alternative mechanism of heavy neutrino pair-production via the fusion of two gluons [24,25] which is relevant to pp supercolliders. As shown in Fig. 7, two gluons from the colliding protons can go via a quark loop into an off-shell Z or Higgs boson which converts into an $N\bar{N}$ pair. The heavy neutrinos, in turn, decay into $\ell(2j)$ and $\ell'(2j)$ final states, say, so that the signal configuration is $\ell\ell'(4j)$. For a Majorana pair, one can have like sign dileptons which with four jets make an almost unique signal for this process. In the case of a Dirac pair and a signal configuration of $\ell^+ e'^-(4j)$, the background from $t\bar{t}$ pair-production and subsequent semileptonic decays of $t, \bar{t}$ would be overwhelming. But now one can hook on to the leptonic decay of one of the W's and search for the signal $\ell^+ e'^- e''^+ (2j)\not{E}_T$ or $\ell^+ e'^- e''^- (2j)\not{E}_T$, where $\not{E}_T$ is the missing transverse energy.

Figure 7: Gluon fusion into decaying heavy neutrino pairs in pp collision

There are several characteristic features of the Z-exchange mechanism:

• The triangular loop has a nonzero contribution only from the axial part of the Z-coupling, the vector part vanishing on account of Furry's theorem.

• The contributing part is an anomaly graph proportional, not only to the third component of the weak isospin of the fermion circulating in the triangle, but also – through the divergence of the axial current – to its mass. Thus the mass difference $|M_U - M_D|$ between the up-type and down-type quarks of the heaviest generation comes in the numerator of the dominant part of the amplitude.

• By Yang's theorem, the amplitude is nonzero only because of the off-shell nature of the Z. The consequent $Q^2 - M_Z^2$ in the numerator cancels the denominator from the propagator making the cross section only weakly dependent on s.

In the light of the above features it turns out that the Higgs-mediated $gg \to H^\star \to N\overline{N}$ cross section is enhanced relative to the Z-mediated one to which it adds incoherently on-account of the difference in the s-channel angular momentum. One can actually have both scalar Higgs H and pseudoscalar Higgs P exchanges (in models with more than one doublets). It has been [27] demonstrated (taking $m_{\rm top} = 160$ GeV) that the cross section in either case is expected to be quite large. More recent works [28] have given detailed discussions of such processes for the specific case when N is a heavy righthanded Majorana neutrino. In this case the discovery limit for N at the forthcoming Large Hadron Collider can go upto $M_N \simeq 10$ TeV.

• SUMMARY

The salient features of current speculations on and searches for very heavy neutrinos N weighing more than 45 GeV can be summarized as follows.

(a) One or more N's of right chirality are needed to implement the seesaw mechanism of generating the mass of a light neutrino. In case the mechanism is effected at the 1-loop level, such N's could have masses $\mathcal{O}\,(10^2$ GeV) and be discovered in the near future.

(b) Several 'reasonable' beyond-standard-model scenarios exist predicting such objects.

(c) If N is stable or long-lived, its mass cannot exceed a few TeV on account of cosmological constraints. However, the mass of a short-lived N is essentially unconstrained by early universe considerations.

(d) Forthcoming collider experiments will seriously probe mass regions $\sim 10^2$ GeV for an unstable N decaying within the detector.

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